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Gravireggeons in extra dimensions and interaction of ultra-high energy cosmic neutrinos with nucleons

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Abstract. We present the results on non-perturbative quantum gravity effects related to extra dimensions which can be comparable, in some cases, with the SM contributions, e.g. in lepton–lepton or lepton–nucleon scattering. The case of cosmic neutrino gravitational interaction with atmospheric nucleons is considered in detail.

1 Gravireggeon effects in multidimensional scattering amplitudes

During the last years there has been a growing practical interest in models with compact extra spatial dimensions. Their compactification radius, R_c, varies from 1 fm to 1 mm, depending on the number of extra dimensions $d = D-4$ [1]. The models predict massive Kaluza–Klein (KK) excitations of the graviton and KK modes of the SM fields (provided the latter are allowed to propagate in higher dimensions). If D-dimensional space-time has a flat metric, the coupling of the massive graviton modes with the SM particle is very weak and is defined by the Newton constant $G_{\rm N} = 1/\bar{M}_{\rm Pl}^2$, where $\bar{M}_{\rm Pl}$ is the reduced Planck mass. Nevertheless, in the case when SM particles are confined to a 4-dimensional flat "brane", summing up the KK graviton excitations results in a D-dimensional gravitational coupling $G_D \sim 1/M_D^{2+d}$, with a fundamental Planck scale M_D of order 1 TeV [1].

Let us first consider the SM in *D-dimensional flat space-time*, $D > 4$, *without gravity*. Due to the extra spatial dimensions, the effective "transverse interaction region" becomes larger than in four dimensions. One manifestation of this is a modification of the Froissart–Martin upper bound in a *flat* space-time with arbitrary D dimensions [2]:

$$\sigma_{\rm tot}^D(s) \leqq {\rm const}(D)\, R_0^{D-2}(s), \qquad (1)$$

$\sqrt{s}$ being the collision energy. The "transverse radius" in (1) is given by $R_0(s) = N(D) \ln s/\sqrt{t_0}$, where t_0 denotes the nearest singularity in the t-channel, assumed non-zero, while $N(D)$ is some integer depending on D. It is interesting to see what happens with the upper bound (1) when we replace infinite extra dimensions by compact ones.

In [3] the Froissart–Martin bound was generalized for scattering in *D-dimensional space-time with compact extra dimensions*. For one extra dimension with the compactification radius R_c, the upper bound is of the form

$$\mathrm{Im}\, T_D(s,0) \leqq {\rm const}(D)\, s\, R_0^{D-2}(s)\, \Phi\left(\frac{R_0}{R_c}, D\right), \qquad (2)$$

where $\mathrm{Im}\, T(s,t)$ is the scattering amplitude, t is the momentum transfer (in D dimensions) and $\Phi(R_0/R_c, D)$ is a known function. At $R_c \ll R_0(s)$ the equality (2) results in [3]

$$\mathrm{Im}\, T_D(s,0) \leqq {\rm const}(D)\, s\, R_0^{D-3}(s)\, R_c, \qquad (3)$$

while in the opposite limit, $R_c \gg R_0(s)$, the inequality (2) reproduces the upper bound (1).

Now let us allow the gravity to come into play. As was argued in a number of papers ($D > 4$) gravity becomes strong in a transplanckian region ($s \gg M_D^2$), since an effective gravitational coupling, $G_D s$, rises with energy.

In [4] the eikonal representation for the scattering amplitude of the gravitons in string theory was obtained:

$$A(s,t) = -2{\rm i}s \int {\rm d}^{D-2}b\, {\rm e}^{{\rm i}qb} \left[{\rm e}^{{\rm i}\chi(s,b)} - 1\right], \qquad (4)$$

where $\chi(s,b) \simeq {\rm i}\mathrm{Im}\, \chi(s,b)$ is large at $b \lesssim b_1 = 2\sqrt{\alpha_g'} \ln s$ (α_g' is the string tension). Thus, one gets asymptotically for the inelastic cross section:

$$\sigma_{\rm in}^D(s) \simeq {\rm const}(D)\, b_1^{D-2}(s). \qquad (5)$$

Due to the absence of infrared divergences in the flat space-time with more than four dimensions, the gravitational cross section (5) appears to be finite and similar to the upper bound (1). The inelastic cross section appears to be finite even in four dimensions in spite of massless exchanges. It is interesting to ask: can this fact be derived rigorously from the general principles?

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In what follows, we will first consider the scattering of two particles in the model with *one* compact extra dimensions ($D = 5$) in the transplanckian kinematical region:

$$\sqrt{s} \gg M_D, \qquad s \gg -t, \tag{6}$$

$t = -q_\perp^2$ being the four-dimensional momentum transfer. The generalization to $D > 5$ is straightforward and will be done below. Thus, we start from a consideration of the scattering of bulk particles in four spatial dimensions, one of which is compactified with the large radius R_c.

In the eikonal approximation the elastic scattering amplitude in the transplanckian kinematical region (6) is given by the sum of reggeized gravitons in the t-channel. We assume that both massless graviton and its KK massive excitations lie on linear Regge trajectories:

$$\alpha(t_D) = \alpha(0) + \alpha'_g\, t_D, \tag{7}$$

where t_D denotes D-dimensional momentum transfer. Since the extra dimension is compact with radius R_c, we come to the splitting of the Regge trajectory (7) into a leading vacuum trajectory:

$$\alpha_0(t) \equiv \alpha_{\mathrm{grav}}(t) = 2 + \alpha'_g t \tag{8}$$

and an infinite sequence of secondary, "KK-charged", gravireggeons [6]:

$$\alpha_n(t) = 2 - \frac{\alpha'_g}{R_\mathrm{c}^2}\, n^2 + \alpha'_g t, \quad n \geqq 1. \tag{9}$$

The string theory implies that the slope of the gravireggeon trajectory is universal for all s, and $\alpha'_g = 1/M_\mathrm{s}^2$, where M_s is the string scale.

If we assume that multidimensional theory at short distances is a string theory, than the scale M_D can be of the order of the fundamental string scale $M_\mathrm{s} = (\alpha'_g)^{-1/2}$. For instance, in the type-I theory of open and closed strings one has [7]

$$M_\mathrm{s} = \left(\frac{g_\mathrm{s}^2}{4\pi}\right)^{2/(2+d)} M_D, \tag{10}$$

where g_s is a gauge coupling at the string scale. This relation leads to a D-dimensional Planck mass a bit higher than the string scale (for $g_\mathrm{s}^2/4\pi \simeq 0.1$).

Thus, instead of taking a "bare" graviton exchange, we calculate a contribution from the trajectory to which this KK graviton mode belongs:

$$-\,G_\mathrm{N}\,\frac{1+\exp(-\mathrm{i}\pi\alpha_n(t))}{\sin\pi\alpha_n(t)}\,\alpha'_g\beta_n^2(t)\left(\frac{s}{s_0}\right)^{\alpha_n(t)}. \tag{11}$$

The Born amplitude is, therefore, of the form

$$A^\mathrm{B}(s,t,n) \tag{12}$$
$$= G_\mathrm{N}(2\pi R_\mathrm{c})\left[\mathrm{i} - \cot\frac{\pi}{2}\alpha_n(t)\right]\alpha'_g\,\beta_n^2(t)\left(\frac{s}{s_0}\right)^{\alpha_n(t)}.$$

In order to get an idea of the possible t-dependence of Regge residues $\beta_n^2(t)$, we consider scattering of D-dimensional gravitons. The corresponding amplitude has been calculated in [4]:

$$A^\mathrm{B}_{\mathrm{string}}(s,t_D) \sim \frac{G_D\, s^2}{|t_D|}\,\frac{\Gamma(1-\alpha'_g t_D/2)}{\Gamma(1+\alpha'_g t_D/2)}\,(\alpha'_g s)^{\alpha'_g t_D}. \tag{13}$$

The expression (13) is valid in the region $\alpha'_g|t| < 1$ in which it can be recast in the form

$$A^\mathrm{B}_{\mathrm{string}}(s,t_D) \sim \frac{G_D\, s^2}{|t_D|}\,\mathrm{e}^{\gamma\alpha'_g t_D}\,(\alpha'_g s)^{\alpha'_g t_D}, \tag{14}$$

where $\gamma \simeq 0.58$ is the Euler constant.

Thus, we have $A(s,t) \sim \exp(\alpha'_g c t)$, where c is of order of unity. Let us assume that the Regge residues in (12) have an analogous t-dependence:

$$\beta_n^2(t) = \beta^2(0)\,\mathrm{e}^{\alpha'_g b_0 (t - n^2/R_\mathrm{c}^2)}. \tag{15}$$

Since the coupling of all KK states to the SM fields is universal in the ADD model, we expect that $\beta_n^2(t)$ depends on n via $t_D = t - n^2/R_\mathrm{c}^2$. Accounting for the fact that the product $\alpha'_g b_0$ appears only in a combination with $\alpha'_g \ln(s/s_0)$, we can neglect it in forthcoming calculations at large s and put $\beta_n^2(t) \simeq \beta^2(0)$. At $t \to 0$, the $n = 0$ expression (11) reproduces the singular term $G_\mathrm{N} s/|t|$ related with the massless graviton, which results in the relation $2\beta^2(0)/\pi s_0^2 = 1$.

The expression for a 5-dimensional eikonal amplitude looks like (k being the exchanged KK quantum number)

$$A(s,t,k) = 2\mathrm{i}R_\mathrm{c} s \int \mathrm{d}^2 b\, \mathrm{e}^{\mathrm{i}q_\perp b + \mathrm{i}k\phi} \int_{-\pi}^{\pi} \mathrm{d}\phi \left[1 - \mathrm{e}^{\mathrm{i}\chi(s,b,\phi)}\right], \tag{16}$$

with the eikonal given by

$$\chi(s,b,\phi) = \frac{1}{4\pi s} \tag{17}$$
$$\times \int_0^\infty q_\perp \mathrm{d}q_\perp\, J_o(q_\perp b)\,\frac{1}{2\pi R_\mathrm{c}} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}n\phi} A^\mathrm{B}(s, -q_\perp^2, n).$$

The variable ϕ runs over the region $-\pi \leqq \phi \leqq \pi$. These inequalities imply that $-\infty \leqq y \leqq \infty$ in the limit $R_\mathrm{c} \to \infty$ (flat extra dimension), $y = R_\mathrm{c}\phi$ being the 5th component of the impact parameter.

One can easily obtain from (16) that at $k = 0$ and $s < 4/R_\mathrm{c}^2$ only modes with $n = 0$ contribute and effectively $\chi(s,b,\phi) = \chi_0(s,b)$, corresponding to the $n = 0$ contribution in the sum in (17). So, at low energy the scattering amplitude does not feel the extra dimensions (the factor R_c is trivial and is absent at proper normalization).

Let us consider first the imaginary part of the eikonal. From (17) and (12) we obtain

$$\mathrm{Im}\,\chi(s,b,\phi) = G_\mathrm{N} s\,\frac{\alpha'_g}{8R_\mathrm{g}^2(s)}\exp\left[-b^2/4R_\mathrm{g}^2(s)\right]\theta_3(\upsilon, q), \tag{18}$$

where

$$R_{\rm g}(s) = \sqrt{\alpha'_g(\ln(s/s_0) + b_0)} \qquad (19)$$

is a "Regge gravitational radius". The quantity θ_3 in (18) is one of the Jacobi θ-functions [8]:

$$\theta_3(\upsilon) = \theta_3(\upsilon, q) = 1 + 2\sum_{n=1}^{\infty} \cos(2\pi n\upsilon)\, q^{n^2}. \qquad (20)$$

In our case, it depends on the variables

$$\upsilon = \frac{\phi}{2\pi}, \quad q = \exp\left[-R_{\rm g}^2(s)/R_{\rm c}^2 \right]. \qquad (21)$$

The function $\theta_3(\upsilon, q)$ is well defined for all (complex) υ and all values of q such as $|q| < 1$. It has a singularity at $q \to 1$ (see below). The θ-functions are often defined in terms of the variable τ:

$$\theta(\upsilon) = \theta(\upsilon|\tau), \qquad (22)$$

where

$$q = e^{i\pi\tau}. \qquad (23)$$

Let us define the ratio

$$a = \frac{R_{\rm c}}{2R_{\rm g}(s)} \qquad (24)$$

(that is, $q = \exp(-1/4a^2)$). From the equality [1]

$$R_{\rm c} = 2 \cdot 10^{31/d-17} \left(\frac{1\,\text{TeV}}{M_D} \right)^{1+2/d} \text{cm} \qquad (25)$$

we see that the compactification scale $R_{\rm c}^{-1}$ varies from 10^{-3} eV for $d = 2$ to 10 MeV for $d = 6$. Since $R_{\rm c}^{-1} \ll (2R_{\rm g}(s))^{-1}$ even at ultra-high energies, we have $a \gg 1$ and, consequently, $(1 - q) \ll 1$.

The behavior of $\theta_3(\upsilon, q)$ at $q \to 1$ can be derived by using the unimodular transformation of the θ_3-function (known also as the imaginary Jacobi transformation) [8]:

$$\theta_3\left(\frac{\upsilon}{\tau} \Big| -\frac{1}{\tau} \right) = (-i\tau)^{1/2} e^{i\pi\upsilon^2/\tau} \theta_3(\upsilon|\tau). \qquad (26)$$

Here $(-i\tau)^{1/2}$ has a principal value which lies in the right half-plane. In the variable q, the equality (26) looks like

$$\theta_3(\upsilon, q) = \left(-\frac{\pi}{\ln q} \right)^{1/2} \sum_{n=-\infty}^{\infty} e^{(2\pi n - \phi)^2/4\ln q}. \qquad (27)$$

The series in the RHS of (27) converges very quickly at $q \to 1$, contrary to the original series (20):

$$\theta_3(\upsilon, q) \qquad (28)$$
$$= 2a\sqrt{\pi} \left\{ e^{-\phi^2 a^2} + \sum_{n=1}^{\infty} \left[e^{-(2\pi n - \phi)^2 a^2} + e^{-(2\pi n + \phi)^2 a^2} \right] \right\}.$$

Notice that $a^2 = -1/4\ln\ q$.

From all that was said above, we get

$$\text{Im}\,\chi(s, b, \phi) \qquad (29)$$
$$\simeq G_{\rm N} s\, \frac{\alpha'_g R_{\rm c} \pi^{1/2}}{8R_{\rm g}^3(s)} \exp\left[-(b^2 + R_{\rm c}^2\phi^2)/4R_{\rm g}^2(s) \right].$$

The expression (18) is directly generalized for d extra dimensions ($d \geqq 1$):

$$\text{Im}\,\chi(s, b, \phi_1, \dots \phi_d) = G_{\rm N} s\, \frac{\alpha'_g}{8R_{\rm g}^2(s)}$$
$$\times \exp\left[-b^2/4R_{\rm g}^2(s) \right] \prod_{i=1}^{d} \theta_3(\upsilon_i, q), \qquad (30)$$

where $\upsilon_i = \phi_i/2\pi$. Correspondingly, we obtain

$$\text{Im}\,\chi(s, b, \phi_1, \dots \phi_d) \simeq G_{\rm N} s\, \frac{\alpha'_g R_{\rm c}^d \pi^{d/2}}{8R_{\rm g}^{2+d}(s)} \qquad (31)$$
$$\times \exp\left[-(b^2 + R_{\rm c}^2\phi_1^2 + \dots + R_{\rm c}^2\phi_d^2)/4R_{\rm g}^2(s) \right].$$

We see from (31) that the imaginary part of the eikonal decreases exponentially in the variables b, ϕ_i outside the region:

$$b^2 + (R_{\rm c}\phi_1)^2 + \dots (R_{\rm c}\phi_d)^2 \lesssim R_0^2(s), \qquad (32)$$

where

$$R_0^2(s) \simeq 4R_{\rm g}^2(s) \ln(s/M_D^2) \qquad (33)$$

at high s.

Let $t_D = (t, -n_1^2/R_{\rm c}^2, \dots, -n_d^2/R_{\rm c}^2)$ be a bulk momentum transfer. Then we get the following expression for the multidimensional scattering amplitude:

$$A_D(s, t, n_1, \dots, n_d)$$
$$= -2is\, R_{\rm c}^d \int d^2b\, e^{iq_\perp b} \int_{-\pi}^{\pi} d\phi_1 \dots \int_{-\pi}^{\pi} d\phi_d$$
$$\times \prod_{i=1}^{d} e^{in_i\phi_i} \left[e^{i\chi(s,b,\phi_1,\dots,\phi_d)} - 1 \right]. \qquad (34)$$

Correspondingly, the inelastic cross section in the space-time with d compact dimensions is given by

$$\sigma_{\rm in}^D(s) = (2\pi R_{\rm c})^d \qquad (35)$$
$$\times \int d^2b \int_{-\pi}^{\pi} d\phi_1 \dots \int_{-\pi}^{\pi} d\phi_d \left[1 - e^{-2\text{Im}\,\chi(s,b,\phi_1,\dots,\phi_d)} \right].$$

As was already shown, the imaginary part of the eikonal is negligibly small outside the region (32). That results in the estimates

$$\sigma_{\rm in}^D(s) \simeq \text{const}(D) \times \begin{cases} R_0^{2+d}(s), & R_{\rm c} \gg R_0(s), \\ R_0^2(s)\, R_{\rm c}^d, & R_{\rm c} \ll R_0(s), \end{cases} \qquad (36)$$

which remind one of the general upper bounds (1) and (3). As was mentioned above, $R_0(s) \ll R_c$ for any reasonable s. So, the size of the compact extra dimensions is irrelevant to the behavior of the inelastic cross section and $\sigma_{in}^D(s) \sim (\alpha'_g)^{D/2-1}(\ln s)^{D-2}$. Only at $s \to \infty$, when the transverse interaction region $R_0(s)$ becomes much larger than R_c, we get $\sigma_{in}^D(s) \sim \alpha'_g R_c^{D-4}(\ln s)^2$.

2 Scattering of the SM fields in the presence of compact extra dimensions

Now we consider the case when *the colliding particles are confined on the 4-dimensional brane*, while the exchange quanta (KK gravitons) are allowed to propagate in the bulk. Thus, the collisions of the SM particles take place in a two-dimensional impact parameter space. In [9,10] the scattering of two SM particles was calculated in the eikonal approximation by summing up only "bare" KK gravitons. The massive graviton modes originating from the extra dimensions change the four-dimensional propagator:

$$\frac{1}{-t} \to \sum_{n_1^2+\dots n_d^2 \geqq 0} \frac{1}{-t + \sum\limits_{i=1}^{d} \dfrac{n_i^2}{R_c^2}}. \quad (37)$$

Since a contribution from only non-reggeized KK excitations of the graviton has been taken into account in [9,10], the eikonal has no imaginary parts in such an approach. As was shown in [10], the D-dimensional brane amplitude has a Born pole at $t = 0$ and an infinite phase. Notice that the series (37) diverges at $d \geqq 2$.

In [11] it was argued that the amplitude of the $M \to N$ transition observed *in four dimensions*, A_{MN}, is related to a corresponding D-dimensional amplitude A_{MN}^D by the relation

$$A_{MN} = (2\pi R_c)^{d(1-(M+N)/2)}\, A_{MN}^D \quad (38)$$

(in our case, $M = N = 2$). The amplitudes A_{MN} have nonzero limit at $R_c \to 0$, reproducing the usual 4-dimensional pseudo-euclidean case. Since the colliding particles are confined on the brane, their momenta lie in four-dimensional space. Therefore, the impact parameter belongs to the two-dimensional space and we have to put $\phi_i = 0$, $i = 1, \dots, d$, in (31). With taking account of this, the expression for the four-dimensional eikonal amplitude (in the presence of d compact extra dimensions) looks like

$$A(s,t) = 2\mathrm{i}s \int \mathrm{d}^2 b\, \mathrm{e}^{\mathrm{i}q_\perp b} \left[1 - \mathrm{e}^{\mathrm{i}\chi(s,b)}\right], \quad (39)$$

where $\chi(s,b) = \chi(s,b,\phi_1 = 0,\dots,\phi_d = 0)$. Taking into account (31), we get the expression

$$\mathrm{Im}\,\chi(s,b) = \frac{1}{\pi^{d/2-1}} \frac{s}{M_D^2} \left(\frac{M_s}{2M_D}\right)^d \left[\ln\left(\frac{s}{s_0}\right)\right]^{-(1+d/2)} \times \exp[-b^2/4R_g^2(s)], \quad (40)$$

where the relation $M_{Pl}^2 = (2\pi R_c)^d\, M_D^{2+d}$ is used [1]. The detailed analysis of the real part of the eikonal will be given elsewhere. Here we only note that, contrary to (40), the real part of the eikonal (with the massless graviton term subtracted) decreases as a power of the impact parameter at large b.

Table 1. Cross sections of the processes induced by graviton exchanges in t-channel (second row) and s-channels (third row) at $\sqrt{s} = 1$ TeV for different numbers of extra dimensions d (in pbarn)

d	2	3	4	5	6
$e^+e^- \to e^+e^- + X$	$1.06 \cdot 10^3$	$1.10 \cdot 10^2$	$1.78 \cdot 10^1$	3.84	1.02
$e^+e^- \to f\bar{f}$	9.3	3.7	2.0	1.3	0.9

The important features of the gravitational contribution to the cross sections are its independence of the types of colliding particles and a strong dependence on the collision energy. So, one can expect that at superplanckian energies gravity exchanges will dominate the SM electroweak interactions. That is why we now focus on leptonic and semileptonic collisions. Gravitational contributions in hadron–hadron collisions are masked at all reasonable energies by the background due to pomeron exchanges.

Let us first consider e^+e^- annihilation. Unfortunately, future linear colliders will provide us with the CMS energies $\sqrt{s}$ around M_D ($0.5 \div 2$ TeV). In order to estimate $\sigma_{in}^{e^+e^-}$ numerically, we need to fix the Regge free parameter s_0 in (40). Since s_0 is related rather with the mass scale of the exchange quanta than with the mass scales of the colliding particles, we can treat the scattering amplitude of the two graviton (13) instead of SM particle collision, and deduce that[1]

$$s_0 = (\alpha'_g)^{-1}. \quad (41)$$

This relation is also motivated by duality [13]. The results of our calculations of the inelastic cross section $\sigma_{in}^{e^+e^-}$ at $\sqrt{s} = 1$ TeV based on the formulae (40) and (41) are presented in the second row of Table 1.

These cross sections are larger than the cross sections of the processes induced by massive graviton exchanges in the s-channel (at least, for $d \leqq 6$).[2] For definiteness, consider the matrix element for fermion pair production $e^+e^- \to f\bar{f}$:

$$\mathcal{M} = G_N T_{\mu\nu}^e P^{\mu\nu\alpha\beta} T_{\alpha\beta}^f \sum_{n_1^2+\dots n_d^2 \geqq 0} \frac{1}{s - \sum\limits_{i=1}^{d} \dfrac{n_i^2}{R_c^2}}. \quad (42)$$

Here $P^{\mu\nu\alpha\beta}$ is the tensor part of the graviton propagator, while $T_{\mu\nu}^{e(f)}$ is the energy-momentum tensor of the field $e(f)$ [17,18]. The sum in (42) diverges for $d \geqq 2$. It can be estimated if one convert it into an integral and introduce

[1] In hadronic physics, the phenomenological parameter $s_0 \approx 1/\alpha'(0)$, where $\alpha'(0) \simeq 1\,\mathrm{GeV}^{-2}$ is the slope of the hadronic Regge trajectories [12].

[2] It is worth to note that, generally, QFTs for $d > 0$ are not renormalizable. So, the following estimates are of illustrative character.

an explicit ultraviolet cut-off M_s. Then we get for $d > 2$:

$$\sum_{n_1^2+\ldots n_d^2 \geqq 0} \frac{1}{s - \sum\limits_{i=1}^{d} \dfrac{n_i^2}{R_\mathrm{c}^2}} \simeq \begin{cases} -\dfrac{2R_\mathrm{c}^d}{(d-2)\Gamma(d/2)(4\pi)^{d/2}}\, M_\mathrm{s}^{d-2}, \ \sqrt{s} \ll M_\mathrm{s}, \\[2ex] \dfrac{R_\mathrm{c}^d}{\Gamma(1+d/2)(4\pi)^{d/2}}\, \dfrac{M_\mathrm{s}^d}{s}, \qquad \sqrt{s} \gg M_\mathrm{s}, \end{cases} \tag{43}$$

(the asymptotics at $\sqrt{s} \ll M_\mathrm{s}$ was first found in [18]).

Taking into account that the sum in indices results in a factor proportional to s^2, we obtain from (41) and (43)

$$\mathcal{M} \sim \lambda\, s^2 \left(\frac{M_\mathrm{s}}{M_D}\right)^{2+d} \times \begin{cases} \dfrac{1}{M_\mathrm{s}^4}, \ \sqrt{s} \ll M_\mathrm{s}, \\[2ex] \dfrac{1}{sM_\mathrm{s}^2}, \ \sqrt{s} \gg M_\mathrm{s}, \end{cases} \tag{44}$$

where $\lambda = \mathrm{O}(1)$ has opposite signs for small and large $\sqrt{s}$. The two asymptotics (44) are well-matched at $\sqrt{s} \simeq M_\mathrm{s}$. Thus, at $\sqrt{s} \gtrsim M_\mathrm{s}$ we arrive at the expression

$$\sigma(e^+e^- \to f\bar{f}) \simeq \lambda^2\, \frac{N_c}{40\pi} \left(\frac{M_\mathrm{s}}{M_D}\right)^{2+d} \frac{s}{M_\mathrm{s}^4}, \tag{45}$$

where N_c represents the number of colors of the final state. The result of the numerical calculations by using formula (45) is presented in the third row of Table 1.

To compare, the hadronic SM background in e^+e^- annihilation ($e^+e^- \to e^+e^- +$ hadrons), including the effects due to the (anti)tagging of the electron and accounting for all available data on $\gamma\gamma$ collisions, was estimated to be [15]

$$\sigma_{\mathrm{had}}^{e^+e^-}(\sqrt{s} = 1\,\mathrm{TeV}) \simeq (2.7\text{–}4.0) \cdot 10^4\,\mathrm{pb}. \tag{46}$$

The SM processes with different final states ($\sum_{q \neq t} q\bar{q}$, W^+W^-, $t\bar{t}$, $\tilde{\chi}^+\tilde{\chi}^-$, $\tilde{\mu}_R^+\tilde{\mu}_R^-$, Zh, etc.) have cross sections which are less than 1 pb at $\sqrt{s} = 1\,\mathrm{TeV}$ (see, for instance, Fig. 1.3.1 from [16]). The highest rate has the process $e^+e^- \to \sum_{q \neq t} q\bar{q}$; its cross section is about 0.7 pb.

Our goal is to find a process in which gravity forces can dominate SM interactions. Such a process has to obey the following requirements:
(i) the colliding energy is much larger than $M_D \simeq 1\,\mathrm{TeV}$;
(ii) the SM cross section does not rise rapidly in s.

The best candidate is the scattering of ultra-high-energy (UHE) neutrinos off the nucleons. These neutrinos are a part of ultra-high-energy cosmic rays (UHECR) with energy $E \gtrsim 10^{18}\,\mathrm{eV}$, which are dominated by extragalactic sources of protons [14]. It is the detection of UHE neutrinos that can help us to discriminate between different origins of UHECR. For instance, in cosmological ("bottom-up") scenarios, neutrino fluxes are almost equal to gamma ray fluxes. In the astrophysical (acceleration) approach, the neutrino flux is only a fraction of the gamma ray flux and is modified due to the propagation of cosmic rays before they reach the Earth.

The cosmic neutrinos with extremely high energies $E \gtrsim 10^{20}\,\mathrm{eV}$ are also believed to explain the so-called Greisen–Zatsepin–Kuzmin (GZK) cut-off of UHECR spectrum [19] (see below). During UHECR propagation, the protons scatter off the cosmic microwave background (CMB):

$$p + \gamma_{\mathrm{CMB}} \to N + \pi. \tag{47}$$

Taking into account that typical CMB photon energies are $10^{-3}\,\mathrm{eV}$, one can see that the nucleon interaction length drops to about 6 Mpc at the GZK bound of $E_{\mathrm{GZK}} \simeq 5 \cdot 10^{19}\,\mathrm{eV}$ [19].[3] The observation of UHECR at $E > E_{\mathrm{GZK}}$ is a serious problem for theories in which the origin of CR is based on the acceleration of charged particles in astrophysical objects. Due to the energy losses (say, through the process (47)), the UHECR particles cannot originate at distances larger than 60 Mpc from the Earth. On the other hand, all potential astrophysical sources of UHECR events are far beyond this distance.

At the same time the process (47) is the origin of so-called cosmogenic neutrinos due to a consequent decay of the charged pion as $\pi^\pm \to \mu^\pm\,\nu_\mu$, $\mu^\pm \to e^\pm\,\nu_e\,\nu_\mu$. The fraction of the proton energy carried by the neutrino is $E_\nu/E_p \approx 0.05$ and is independent of E_p. The cosmogenic neutrino flux was first estimated in [21,22]. More recent estimates can be found in [23–26]. The predicted fluxes depend on the evolution parameter m and on the value of the redshift z, and lie in the range: $E_\nu^2\,\Phi_\nu \simeq (0.5 \cdot 10^{-9}\text{–}10^{-8})\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ at $E_\nu = 10^{20}\,\mathrm{eV}$ ($\nu = \nu_\mu, \bar{\nu}_\mu, \nu_e$).[4]

There are, however, other possible origins for the UHE neutrinos. It is usually anticipated that $\Phi_{\nu_e} \approx \Phi_{\bar{\nu}_\mu} \approx \Phi_{\nu_\mu}$. We present below the total flux of muonic neutrinos and antineutrinos in a number of models at $E_\nu = 10^{20}\,\mathrm{eV}$. In active galactic nuclei (AGN), the dominant mechanism for neutrino creation is the accelerated proton energy loss due to pp or $p\gamma$ interactions [27]. Note that AGN produce a large fraction of the gamma rays in the Universe, and their spectra agree with the prediction that gamma rays are produced by hadrons. In the AGN approach it was obtained that $E_\nu^2\,\Phi_\nu \simeq 0.3 \cdot 10^{-8}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ [27]. In the Z-burst scenario, cosmic neutrinos with extremely high energies ($E_\nu > 4 \cdot 10^{21}(1\,\mathrm{eV}/m_\nu)\,\mathrm{eV}$) collide with relic neutrinos [28, 29]. If the masses of the background neutrinos m_ν are of several eV, the cosmic neutrinos initiate high-energy particle cascades which can contribute 10% to the observed cosmic ray flux at energies above the GKZ cut-off (one of the main processes is a resonant $\nu\nu$ collision via the Z-boson). The neutrino flux is $E_\nu^2\,\Phi_\nu \simeq 0.3 \cdot 10^{-6}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ [28]. In the so-called topological defect models [30], UHECR are produced via

[3] Below E_{GZK}, the dominant energy loss for the proton is due to the process $p\,\gamma_{\mathrm{CMB}} \to p\,e^+e^-$, down to the threshold energy of $4.8 \cdot 10^{17}\,\mathrm{eV}$.

[4] Some cosmic ray protons with energies above $10^{20}\,\mathrm{eV}$ are converted into neutrons by pion photo-production. The neutrons decay again into protons during their propagation producing electronic anti-neutrinos. This mechanism is important at $E_{\nu_e} \lesssim 10^{17}\,\mathrm{eV}$.

decays of supermassive X-particles related to a grand unification theory. The expected neutrino flux is about $E_\nu^2 \Phi_\nu \simeq 0.5 \cdot 10^{-6}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ [30]. In the gamma ray bursts (GRB) model [31], the neutrino flux is strongly suppressed at $E_\nu > 10^{19}$ eV, since the protons are not expected to be accelerated to energies much larger than 10^{20} eV.

It is worth to mention model-independent upper bounds on the intensity of high-energy neutrinos produced by photo-meson interactions. If the size of the cosmic ray source is not larger than the photo-meson free path, the upper limit is (for evolving sources) $4.5 \cdot 10^{-8}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ [32]. However, for optically thick pion photo-production sources, the upper limit is less restrictive: $2.5 \cdot 10^{-6}\,\mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$ [33]. Note that the considerably higher flux of cosmogenic neutrinos was obtained in [34]. The cosmogenic flux is the most reliable, as it relies only on two assumptions:
(i) the observed extremely high-energy cosmic rays (EHECR) contain nucleons;
(ii) EHECR are primarily extragalactic in origin.

One possible way to resolve the GZK puzzle[5] is to assume that the primary UHECR particles are neutrinos which deposit a part of their energy to proton fragments in νN interactions. Unfortunately, the SM neutrino–nucleon cross sections are not large enough to resolve the problem. Indeed, at $10^{16}\,\mathrm{eV} \lesssim E_\nu \lesssim 10^{21}$ eV the conventional contributions from charged and neutral current νN-scattering can be parameterized by [35]

$$\sigma_{\nu N}^{cc} \simeq 4.429 \cdot 10^3 \left(\frac{E_\nu}{10^8\,\mathrm{GeV}} \right)^{0.363} \mathrm{pb},$$
$$\sigma_{\nu N}^{nc} \simeq 1.844 \cdot 10^3 \left(\frac{E_\nu}{10^8\,\mathrm{GeV}} \right)^{0.363} \mathrm{pb}. \qquad (48)$$

The total SM cross section for $\bar{\nu} N$-scattering has practically the same magnitude and energy dependence at the energies under consideration [35]. Putting $E_\nu = 10^{21}$ eV in (48), we get the estimate $\sigma_{SM}^{\nu N} \simeq 3.55 \cdot 10^5$ pb. Such a value of the neutrino–nucleon cross section is too small to be relevant to the GZK problem.

So, interactions beyond the SM[6] are needed in order to explain the possible excess of the UHECR flux. One possibility is high-energy scattering mediated by gravitational forces in theories with compact extra dimensions [36–40]. In a number of papers [40–47] it was shown that in a model with extra dimensions the neutrino–nucleon cross section can be enhanced by black hole production. The corresponding cross sections were estimated to be one order of magnitude or more above the SM predictions (48) at $E_\nu \gtrsim 10^{18}$ eV.

In [41, 43] the opportunities were considered to search for black hole signatures by using neutrino telescopes such as AMANDA/IceCube, Baikal, ANTARES or NESTOR. The expected black hole production cross section is around 10^6 pb for $M_{\mathrm{BH}}^{\min} = M_D = 1$ TeV, where $M_{\mathrm{BH}}^{\min}$ is a minimal mass of the produced black hole.[7]

Another possibility, which we will concentrate on, is the observation of air showers triggered by UHE neutrino interactions. The technique used for studying extensive air showers of UHECRs or UHE neutrinos is the detection of shower particles by ground detectors, or the detection of fluorescence light produced by the shower. The first technique was used by one of the largest AGASA experiment, while the second one was developed for a Fly's Eye (HiRes) detector. The largest project under construction is the Pierre Auger Observatory [48]. It will consist of two sites, each having 1600 particle detectors overlooked by four fluorescence detectors. For a detailed study of extensive air showers with energy above 10^{18} eV, 10% of the events will be detected by both ground array and fluorescence detectors.

It is also worth to mention the space-based experiments EUSO and OWL which will be sensitive to CRs with energies above 10^{19} eV. The future of neutrino astronomy may be related with radio frequency detectors, such as RICE and ANITA.

The neutrino interaction length is given by (in units of km water equivalent, by 1 km we take $\equiv 10^5\,\mathrm{g\,cm^{-2}}$)

$$L_\nu(E_\nu) \simeq 1.7 \cdot 10^7 \left[\frac{1\,\mathrm{pb}}{\sigma_{\nu N}(E_\nu)} \right] \mathrm{km\ we}. \qquad (49)$$

For a typical black hole production cross section $\sigma_{\nu N} = 10^6$ pb, we get $L_\nu = 17$ km we. This interaction length is much larger than the vertical Earth's atmospheric depth, which is equal to 0.01 km we. The atmospheric depth for neutrinos transverse (almost) horizontally is 36 times larger. That is why it was proposed to search for uniformly produced quasi-horizontal showers at ground level [49].[8]

The Fly's Eye and AGASA Collaborations have searched for deeply penetrating quasi-horizontal air showers, with a depth $L_{\mathrm{sh}} > 2500\,\mathrm{g\,cm^{-2}}$. The probability of cosmic protons and gamma rays initiating air showers deeper than $2500\,\mathrm{g\,cm^{-2}}$ is about 10^{-9}. Thus, any shower starting that deep in the atmosphere is a nice candidate for a neutrino event.

The non-observation of such events puts an upper limit on the product of the neutrino differential flux, $\Phi_\nu = (1/4\pi)\mathrm{d}N_\nu/\mathrm{d}E_\nu$, times the neutrino–nucleon cross section. The Fly's Eye Collaboration gives the bound which can be parametrized by [50]

[5] Note, however, the recent paper in [20], in which it is argued that the data from the Fly's Eye, HiRes and Yakutsk cosmic ray experiments are consistent with the expected suppression of cosmic ray spectrum above $5 \cdot 10^{19}$ eV. The AGASA data show an excess in this region.

[6] There is, however, a possibility that SM instanton-induced processes may give a large neutrino–nucleon cross section [39].

[7] The production rate of black holes depends on the number of extra dimensions and, essentially, on the ratio $M_{\mathrm{BH}}^{\min}/M_D$.

[8] At large zenith angles the background from hadronic cosmic rays is negligible, since the showers initiated by hadrons are high in the atmosphere due to the very short interaction length of the proton. Around 10^{20} eV, the hadronic mean free path is only $40\,\mathrm{g\,cm^{-2}}$, and gamma rays of such energy have interactions lengths of 45–$60\,\mathrm{g\,cm^{-2}}$.

$$(\Phi_\nu\, \sigma_{\nu N})(E_\nu) \le 3.74 \cdot 10^{-42} \left(\frac{E_\nu}{1\,\text{GeV}}\right)^{-1.48} \text{GeV}^{-1}\,\text{s}^{-1}\,\text{sr}^{-1}, \quad (50)$$

while the upper limit from [38] can be recast as follows:

$$(\Phi_\nu\, \sigma_{\nu N})(E_\nu) \le 10^{-41}\, \bar{y}^{-1/2} \left(\frac{E_\nu}{1\,\text{GeV}}\right)^{-1.5} \text{GeV}^{-1}\,\text{s}^{-1}\,\text{sr}^{-1}, \quad (51)$$

where $\bar{y}$ is the average fraction of the neutrino's energy deposited into the shower. The inequalities are valid in the range 10^8 GeV$\le E_\nu \le 10^{11}$ GeV, *provided* $\sigma_{\nu N}(E_\nu) \le 10\,\mu$b.

Let us now estimate the neutrino–nucleon cross section in our approach. The neutrino scatters off the quarks and gluons distributed inside the nucleon (see the comment after formula (53)). Then the cross section is presented by

$$\sigma_{\text{in}}^{\nu N}(s) = \int\limits_{x_{\min}}^{1} \mathrm{d}x \sum_i f_i(x, \mu^2) \sigma_{\text{in}}(\hat{s}), \quad (52)$$

where $f_i(x, \mu^2)$ is the distribution of parton i in momentum fraction x, and $\hat{s} = xs$ is the invariant energy of the partonic subprocess. In our approach, the partonic cross section $\sigma_{\text{in}}(\hat{s})$ is defined via the eikonal (40). As it follows from (40), $\chi(\hat{s}, b)$ is small at $\hat{s} \lesssim M_D^2$, and we can put $x_{\min} = M_D^2/s$ in (52). At $\hat{s} \geqq M_D^2$, the main contribution comes from the region

$$b^2 \lesssim b_{\max}^2(\sqrt{\hat{s}}) = 4R_{\text{g}}(\hat{s})\,[\ln(\hat{s}/M_D^2) + 1]. \quad (53)$$

We choose the neutrino energy E_ν to be 10^{17} eV, 10^{18} eV, 10^{19} eV, 10^{20} eV, and 10^{21} eV. The invariant energy of the νN collision is then 14 TeV, 43 TeV, 137 TeV, 433 TeV and 1370 TeV, respectively. Since $b_{\max}(\sqrt{s} = 1370\,\text{TeV}) \simeq 3 \cdot 10^{-2}\,\text{GeV}^{-1} = 6 \cdot 10^{-3}$ fm (for $2 \leqq d \leqq 6$), our assumption that the neutrino interacts with the proton constituents is well justified.

We use the set of parton distribution functions (PDFs) from [51] based on an analysis of existing deep inelastic data in the next-to-leading order QCD approximation in the fixed-flavor-number scheme. The extraction of the PDFs is performed simultaneously with the value of the strong coupling and high-twist contributions to structure functions. The PDFs are available in the region $10^{-7} < x < 1$, $2.5\,\text{GeV}^2 < Q^2 < 5.6 \cdot 10^7\,\text{GeV}^2$ [51]. We take the mass scale in PDFs to be $\mu = 1/b_{\max}(\sqrt{\hat{s}})$, with $b_{\max}$ defined by (53). The result of our calculations of $\sigma_{\text{in}}^{\nu N}(E_\nu)$ is presented in Table 2.[9] These neutrino–nucleon cross sections do not violate the experimental upper bounds (50) and (51).

Note that the total SM cross sections for $(\nu + \bar{\nu})$-scattering defined by formula (48) are equal to $6.27 \cdot 10^3$ pb, $1.45 \cdot 10^4$ pb, $3.35 \cdot 10^4$ pb, $7.72 \cdot 10^4$ pb, and $1.78 \cdot 10^5$ pb, respectively. Thus, the SM interactions become comparable with (larger than) the gravity interaction for $d = 3 \div 4$ (for $d \geqq 4 \div 5$), depending on the neutrino energy E_ν.

Table 2. Inelastic neutrino–nucleon cross section for the graviton-induced scattering at fixed neutrino energy, E_ν, for different numbers of extra dimensions d (in pbarn)

d	2	3	4	5	6
E_ν=10^{17} eV	$8.63\cdot10^4$	$5.63\cdot10^3$	$5.53\cdot10^2$	$7.16\cdot10^1$	$1.13\cdot10^1$
E_ν=10^{18} eV	$6.53\cdot10^5$	$3.39\cdot10^4$	$2.47\cdot10^3$	$2.26\cdot10^2$	$2.41\cdot10^1$
E_ν=10^{19} eV	$4.20\cdot10^6$	$2.05\cdot10^5$	$1.21\cdot10^4$	$8.59\cdot10^2$	$6.99\cdot10^1$
E_ν=10^{20} eV	$2.05\cdot10^7$	$1.32\cdot10^6$	$7.06\cdot10^4$	$4.29\cdot10^3$	$2.94\cdot10^2$
E_ν=10^{21} eV	$8.74\cdot10^7$	$7.47\cdot10^6$	$4.56\cdot10^5$	$2.52\cdot10^4$	$1.52\cdot10^3$

The number of horizontal hadronic air showers with the energy E_{sh} larger than a threshold energy E_{th}, initiated by the neutrino–nucleon interactions, is given by

$$\begin{aligned} N_{\text{sh}}(E_{\text{sh}} \geqq E_{\text{th}}) &= TN_A \Big[\int \mathrm{d}E_\nu\, \Phi_\nu(E_\nu)\, \sigma_{\nu N}^{\text{grav}}(E_\nu)\, \mathcal{A}(E_\nu)\, \theta(E_\nu - E_{\text{th}}) \\ &+ \sum_{i=e,\,\mu,\,\tau} \int \mathrm{d}E_{\nu_i}\, \Phi_{\nu_i}(E_{\nu_i})\, \sigma_{\nu_i N}^{SM}(E_{\nu_i})\, \mathcal{A}(y_i E_{\nu_i}) \\ &\times \theta(y_i E_{\nu_i} - E_{\text{th}}) \Big], \end{aligned} \quad (54)$$

where $N_A = 6.022 \cdot 10^{23}\text{g}^{-1}$, T is a time interval, and $\mathcal{A}$ is the detector acceptance (in units of km^3 steradian water equivalent). The quantity $\Phi_{\nu_i}(E_{\nu_i})$ in (54) is the flux of the neutrino of type i, and $\Phi_\nu(E_\nu) = \sum_{i=e,\,\mu,\,\tau} \Phi_{\nu_i}(E_{\nu_i})$.[10] The inelasticity y_i defines a fraction of the neutrino energy deposited into the shower in the corresponding SM process (see below).

The AGASA acceptance for deeply penetrating quasi-horizontal air showers with zenith angles $\theta > 75°$ can be found in the second paper of [46]. It rises linearly in E_{sh} in the interval $10^7\,\text{GeV} < E_{\text{sh}} < 10^{10}\,\text{GeV}$, while in the ultra-high-energy region the acceptance is constant and equal to $\mathcal{A}(E_{\text{sh}} \geqq 10^{10}\,\text{GeV}) \approx 1.0\,\text{km}^3$ we sr [46].

The neutrino acceptance of the Pierre Auger detector is roughly 30 times larger, taking into account the ratio between Auger and AGASA surface areas. The acceptance of the Auger ground surface array has been studied in detail in [52], while the acceptance of the fluorescence detector to neutrino-like air showers with large zenith angles was calculated in [54, 55]. The Auger observatory efficiency is high, since the low target density in the atmosphere is compensated by the very large surface area of the array (each side of it covers an area of 3000 km^2). The highest efficiency for quasi-horizontal shower detection is expected at $E_{\text{sh}} > 10^9$ GeV [52].

The number of extensive quasi-horizontal showers induced by neutrinos with energy larger than some threshold energy E_{th}, which can be detected by the array of the southern site of the Pierre Auger observatory, is presented

[9] The SM contributions to the neutrino–nucleon cross sections are not included in Table 2.

[10] Both neutrino and antineutrino are everywhere included in the sum.

Table 3. Yearly event rates for nearly horizontal neutrino-induced showers with $\theta_{\text{zenith}} > 70^\circ$ and $E_{\text{sh}} \geqq E_{\text{th}}$ for cosmogenic neutrino flux from [25] at three values of threshold energy E_{th}. The number of events corresponds to one side of the Auger ground array

d	2	3	4	5	6
$E_{\text{th}} = 10^8$ GeV	34.88	2.00	0.32	0.21	0.20
$E_{\text{th}} = 10^9$ GeV	30.21	1.66	0.21	0.12	0.12
$E_{\text{th}} = 10^{10}$ GeV	13.16	0.74	0.062	0.025	0.022

in Table 3. The neutrino–nucleon cross section $\sigma_{\nu N}^{\text{grav}}$ in (54) describes the contributions from the reggeized KK gravitons. The cosmogenic neutrino flux is from [25], assuming a maximum energy of $E_{\max} = 10^{21}$ eV for the UHECR. The acceptance of the Auger detector is taken from [52] (it is not assumed that the shower axis falls certainly in the array).

The neutrino–nucleon inelastic interactions induced by gravireggeons remind us of the SM neutral currents events. We assume that such events should result in hadronic dominated showers without leading lepton. That is why we choose the inelasticity to be equal to unity[11] for the events induced by gravireggeon exchange (the first term in the RHS of (54)). We have also put $y_e = 1$ for the SM *charged* current interactions initiated by electronic neutrino, while for the SM *neutral* interactions initiated by ν_e and for ν_μ/ν_τ events we have taken $y_e = y_\mu = y_\tau = 0.24$, following the calculations presented in [53].

The neutrino event rates are expected to be much higher for the neutrino fluxes obtained in "optimistic" scenarios considered in [34]. As an example, we have presented the yearly event rates for the Z-burst scenario in Table 4. One can see from Table 4 that the main contribution to the shower rate comes from the region of extremely high neutrino energies ($E_\nu > 10^{10}$ GeV). This can be understood as follows: at UHEs, the neutrino flux times E_ν varies slowly in E_ν in the Z-burst model (up to $2.5 \cdot 10^{12}$ GeV), while both the acceptance of the Auger array and the "gravitational" part of the neutrino–nucleon cross section rise with the neutrino energy (see Table 2).

The calculations of the yearly event rates in the energy interval 10^8 GeV $\leqq E_{\text{sh}} \leqq 10^{11}$ GeV in the Z-burst scenario result in 44, 2.7, 0.38, 0.26, and 0.25 for $d = 2, 3, 4, 5$ and 6, respectively. Remembering that the combined results from AGASA and Fly's Eye imply an upper bound of 3.5 at 90% CL from quasi-horizontal neutrino events [46], and taking into account that the AGASA acceptance is roughly 30 times smaller than the Auger acceptance, we can conclude that the Z-burst neutrinos do not violate the bounds (50) and (51) in our scheme for $d \geqq 3$.[12]

Table 4. The same as in Table 3, but for the Z-burst neutrino flux from [28]

d	2	3	4	5	6
$E_{\text{th}} = 10^8$ GeV	$12.60 \cdot 10^2$	$11.53 \cdot 10^1$	9.26	1.90	1.50
$E_{\text{th}} = 10^9$ GeV	$12.59 \cdot 10^2$	$11.53 \cdot 10^1$	9.26	1.90	1.50
$E_{\text{th}} = 10^{10}$ GeV	$12.55 \cdot 10^2$	$11.51 \cdot 10^1$	9.20	1.85	1.44

[11] The estimates from [40] are not applicable in our case, since in [40] the energy loss in the *elastic* neutrino–nucleon cross section induced by "bare" gravitons was considered, while we deal with *inelastic* cross section in the gravireggeon model.

3 Conclusions

In the model with compact extra spatial dimensions, we have calculated the contribution of the KK gravireggeons to the inelastic cross section of the high-energy scattering of both D-dimensional and four-dimensional SM particles. The usually adopted summing of non-reggeized gravitons leads to a divergent sum in the KK-number n (for $D \geqq 6$). In our approach, on the contrary, the contribution of the gravireggeon with the KK-number n to the eikonal is exponentially suppressed at large n. As a result, the corresponding sum in n is finite, and it can be analytically calculated.

In the case when the SM fields propagate in all D dimensions, the dependence of the inelastic cross section on the invariant energy $\sqrt{s}$ appeared to be similar to the upper limit for the total cross section obtained previously for the SM in the D-dimensional flat space-time without gravity. When, on the contrary, only gravity lives in extra dimensions, the imaginary part of the eikonal is derived in a closed form, which depends (except for $\sqrt{s}$ and the impact parameter b) on the number of extra dimensions $d = D - 4$ and their size R_{c}.

We have estimated the event rate for the quasi-horizontal air showers, induced by the interactions of UHE neutrinos with nucleons, which can be yearly detected by the ground array of the Pierre Auger observatory. It decreases rapidly if d varies from 2 to 5. For $d = 4$, we expect 10 events per year for the neutrino flux predicted in the Z-burst model. For the cosmogenic neutrino flux, gravireggeon-induced interactions do not increase the event rate significantly with respect to the number of the neutrino events calculated in the SM, except for the case $d \leqq 3$, which is likely to be excluded by the cosmological data.

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[12] We do not discuss here *cosmological bounds* on the number of extra dimensions [1] (see also [56] and references therein).

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